Vehicle Routing Problem with Packing Constraints

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Abstract

Vehicle routing problem is defined as determining routes for vehicles serving to a set of customers while minimizing costs. This definition, which ignores packing conditions in transportation of physical units, lacks of validity of the results. Classical vehicle routing models don't ensure packing of the loads, since load dimensions are not taken into consideration. Solutions obtained from such models are subject to fail because of probable dimensional incompatibility of the loads. In this paper, a new model is developed in order to overcome this deficiency and its results are examined.

Keywords: Routing, Packing, Logistics.

Introduction

In vehicle routing models, a general expression neglecting the dimensions of the loads has been used as capacity constraints until now. However, such an expression becomes insufficient when dealing with physical units in daily life applications. Routes obtained by a classical capacity expression may lose their validity because of dimensional incompatibility of the loads, even if the capacity of a vehicle is sufficient by total weight or volume. For this reason, it is needed to develop a comprehensive model which includes packing constraints.

Classical Vehicle Routing Approach

Vehicle Routing Problem (VRP) is one of the hardest problems in combinatorial optimization with its *NP-Hard* structure. A vast of literature exists on the problem which attracted intensive attention either academically or commercially. The roots of the problem come from well known "Traveling Salesman Problem". Kulkarni and Bhave [11] gave some integer formulations of the both problems. Laporte [12] surveyed on exact and approximate methods for the vehicle routing problem and presented some models in the literature. Exact methods developed for VRP are presented by Toth and Vigo [16] with various VRP formulations. Kohl

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and Madsen [10] reported that they solved some of the Solomon [14] test problems optimally by langrangian relaxation up to 100 customers. Fisher *et al.* [7] also obtained optimal results by *k*-tree relaxation besides langrangian relaxation. Among the studies concerning heuristic solution techniques Laporte *et al*. [13], Breedam [3] and Gendrau *et al*. [8] can be seen.

To make a definition, with graphical notations let $V = \{v_0, v_1, ..., v_n\}$ be the set of vertices, $A = \{(v_i, v_j) \mid v_i, v_j \in V; i \neq j\}$ the set of arcs and $C = (c_{ij})$ a cost matrix defined on *A*. The vehicle routing problem concerns assignment of vehicles leaving from depot (represented by "v_0") to all other points in *V*, while minimizing costs. Having $V' = V - \{v_0\}$ and $|V'| = n$, linear model of VRP can be obtained as follows.

$$\psi_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels from } i \text{ to } j, \\ 0, & \text{else.} \end{cases}$$

$$\min \sum_i \sum_j \sum_k c_{ij} \cdot \psi_{ijk} \tag{1}$$

$$\sum_{j \in V} \sum_k \psi_{ijk} = 1 \quad \forall i \in V' \tag{2}$$

$$\sum_{i \in V'} \psi_{i0k} \leq 1 \quad \forall k \tag{3}$$

$$\sum_{i \in V} \psi_{ihk} - \sum_{j \in V} \psi_{hjk} = 0 \quad \forall h \in V, k \tag{4}$$

$$\sum_{j \in V'} \sum_k \psi_{0jk} \geq 1 \tag{5}$$

$$\sum_{i \in V'} \sum_{j \in V} d_i \cdot \psi_{ijk} \leq Q_k \quad \forall k \tag{6}$$

$$u_i - u_j + n \cdot \sum_k \psi_{ijk} \leq n - 1 \quad \forall i, j \in V' \tag{7}$$

$$\psi_{ijk} \in \{0,1\} \quad \forall\, i, j, k \tag{8}$$

$$u_i \geq 0 \quad \text{and integer} \quad \forall i \in V' \tag{9}$$

In the model, (1) shows goal function. (2) states that all customers must be visited exactly once. According to (3), a vehicle can not leave the depot more than once. (4) is the flow equation on the network. (5) ensures that at least one vehicle will leave the depot to serve the customers. (6) prevents exceeding the vehicle capacities either by volume or weight. With (7), subtours are eliminated. Variable definitions are given in (8) and (9).

Solutions obtained from this model ensure that the vehicle capacities are not exceeded, but it can not be ensured that the loads can be packed into the vehicle containers since the physical dimensions are not taken into consideration. If the physical dimensions of the loads are not compatible to be packed, routes obtained from the solutions will lose their validity. This situation can be explained by a small example as the following:

It is needed to plan routes of an operation where loads of nine cities will be collected and brought back to depot. The firm has two vehicles to perform this operation, having dimensions "100 x 100 x 100 dm^3" and "90 x 80 x 60 dm^3". Volumes of these vehicles are calculated as "1000 m^3" and "432 m^3" respectively. Load dimensions of the cities and their volumes are given in Table 1. The cities are represented by letters. Geographical distribution of these cities can be seen in "Figure 1_a)".

Table 1. Loads of the cities for the problem.

City	L (dm)	W (dm)	H (dm)	Vol. (m^3)
A	90	80	80	**576**
B	50	40	25	**50**
C	100	80	10	**80**
D	100	60	20	**120**
E	90	30	20	**54**
F	60	40	30	**72**
G	60	50	40	**120**
H	50	40	30	**60**
I	70	50	20	**70**

Having these data, the firm will solve the problem by given model of the vehicle routing problem. The result can be seen in "Figure 1_b)". Aggregated route, capacity and load information are given in Table 2.

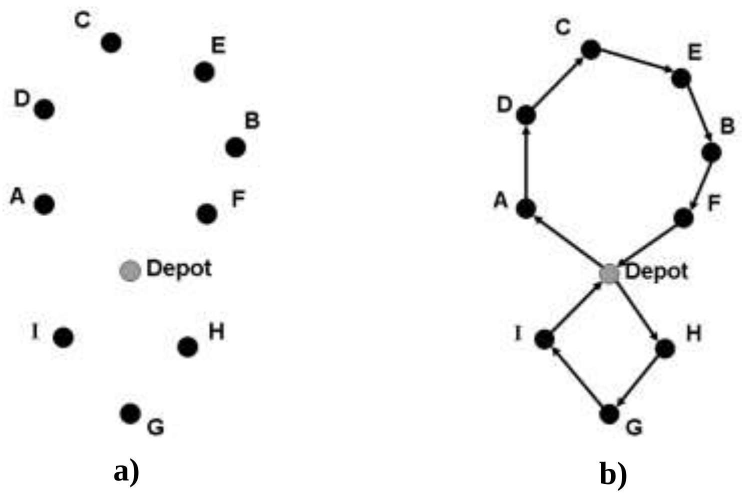

Figure 1. a) Geographical distribution of the cities **b)** Solution of VRP model.

Table 2. Solution information for the example problem.

Vehicle	Route	Capacity (m^3)	Load (m^3)
1	"A–D–C–E–B–F"	1000	952
2	"H–G–I"	432	250

It seems normal that the sum of the loads of the first vehicle is "952 m^3 < 1000 m^3". There is even an excess capacity. The vehicle will not come across to any problem at the first four stops. Loading scheme of the vehicle will most probably be like in Figure 2, when it arrives at the fifth city "*B*" after collecting the load of city "*E*". With this scheme, a problem will be encountered in loading the load of city "*B*" having dimensions "50 x 40 x 25 dm^3". The smallest dimension of free space remained in the vehicle is "10 dm". It will not be possible to pack the load of city "*B*" and similarly the load of city "*F*" which has dimensions "60 x 40 x 30 dm^3". Consequently, the vehicle will have to return back to depot without collecting the loads of these two cities in the route. It is clear that how costly this will be to the firm. Moreover, the classical vehicle routing model will lose its validity since the firm can no more rely on the routes obtained by that model. To overcome this deficiency, it is needed to develop a new model including packing conditions. "*Packing Problem*" has to be analyzed for this.

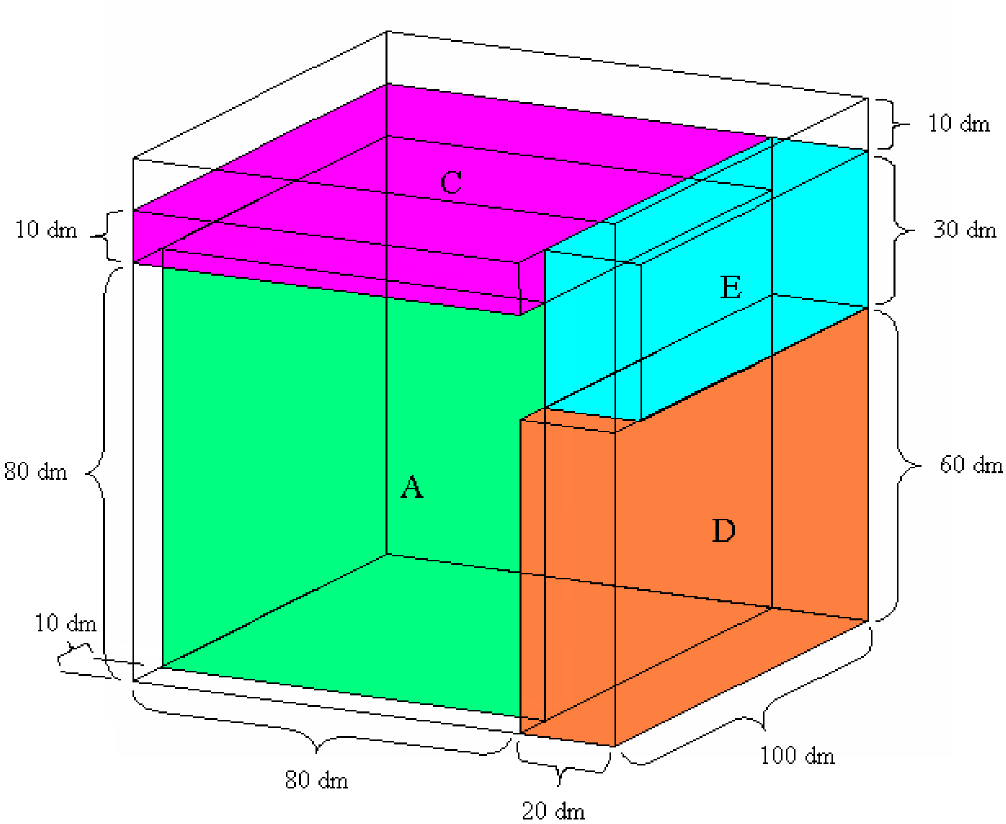

Figure 2. Loading scheme of the first vehicle when it arrives at city "*B*".

Packing Problem

Packing problem can be defined as loading of orthogonal loads optimally into a container. According to Dyckhoff's [6] classification, the problem can be expressed as "3/V/I/", "3/B/O/" or "3/V/D/". Bischoff and Ratcliff [2] studied some practical aspects of the problem. Although the problem has a three dimensional structure, it is highly correlated with two dimensional "*cutting problem*". Various packing problems are examined and the relation of the problem with the cutting problem is put forward by Dowsland and Dowsland [5]. The studies in the literature for *3D* packing problem are generally based on the reduction of the problem to *2D* cutting problem. Studies of Terno *et al*. [15] and Young and Maing [17] can be given as examples of this approach. There also exist *3D* approaches such as the study of Han *et al.* [9], which uses a dynamic programming approach concerning *L*-shaped patterns for loading. Some mathematical models for the packing problem can be found in studies of Chen *et al*. [4] and Ballew [1].

Mathematical Model for Vehicle Routing Problem with Packing Constraints

Vehicle routing problem structurally requires a multi-vehicle fleet for operations. For this reason it must be noted that there will be more than one container in a model for "*Vehicle Routing Problem with Packing Constraints* (VRPPC)". Since distribution operations are generally performed with heterogeneous fleets, different dimensional containers should also be taken into consideration. Consequently, the packing problem to be handled should be in the form of "A selection of objects and all items" as *assignment* and "Different figures" as *assortment* according to Dyckhoff´s [6] classification. The packing model given by Chen *et al*. [4] is compatible with this situation by its "3/V/D/ " structure. Using the packing approach in their study (constraints 17 through 26 and 28 through 33), VRPPC model is formulated as below:

Variables

ψ_{ijk} : "1" if vehicle "k" travels from "i" to "j"; "0" else.

w_i : Total weight of the boxes belonging to city "i".

Q_k : Weight capacity of the vehicle "k".

u_i : An integer variable.

nm : Number of cities.

$s_{\alpha k}$: "1" if box "α" is transported in vehicle "k"; "0" else.

K_i : The set of boxes belonging to city "i".

n_i : Number of boxes belonging to city "*i*".

p_α, q_α, r_α : *Length, width* and *height* of box "α".

L_k, W_k, H_k : Container *length, width* and *height* of vehicle "*k*".

x_α, y_α, z_α : FLB (Front-Left-Bottom) coordinates of box "α".

$a_{\alpha\beta}$: "1" if box "α" is in left of box "β"; "0" else.

$b_{\alpha\beta}$: "1" if box "α" is in right of box "β"; "0" else.

$c_{\alpha\beta}$: "1" if box "α" is in front of box "β"; "0" else.

$d_{\alpha\beta}$: "1" if box "α" is in behind of box "β"; "0" else.

$e_{\alpha\beta}$: "1" if box "α" is in below of box "β"; "0" else.

$f_{\alpha\beta}$: "1" if box "α" is in above of box "β"; "0" else.

$lx_\alpha, ly_\alpha, lz_\alpha$: "1" if length of box "$\alpha$" is parallel to "x", "y" or "z" respectively; "0" else.

$wx_\alpha, wy_\alpha wz_\alpha$: "1" if width of box "$\alpha$" is parallel to "x", "y" or "z" respectively; "0" else.

$hx_\alpha, hy_\alpha, hz_\alpha$: "1" if height of box "$\alpha$" is parallel to "x", "y" or "z" respectively; "0" else.

Linear Model

$$\min \ \mu \cdot \left(\sum_i \sum_j \sum_k c_{ij} \cdot \psi_{ijk} \right) + \gamma \cdot \left(\sum_j \sum_k L_k \cdot W_k \cdot H_k \cdot \psi_{0jk} - \sum_\alpha p_\alpha \cdot q_\alpha \cdot r_\alpha \right) \quad (10)$$

$$\sum_{j \in V} \sum_k \psi_{ijk} = 1 \quad \forall i \in V' \quad (11)$$

$$\sum_{i \in V'} \psi_{i0k} \le 1 \quad \forall k \quad (12)$$

$$\sum_{i \in V} \psi_{ihk} - \sum_{j \in V} \psi_{hjk} = 0 \quad \forall h \in V, k \quad (13)$$

$$\sum_{j \in V'} \sum_k \psi_{0jk} \ge 1 \quad (14)$$

$$\sum_{i \in V'} \sum_{j \in V} w_i \cdot \psi_{ijk} \le Q_k \quad \forall k \quad (15)$$

$$u_i - u_j + nm \cdot \sum_k \psi_{ijk} \le nm - 1 \quad \forall i, j \in V' \quad (16)$$

$$x_\alpha + p_\alpha \cdot lx_\alpha + q_\alpha \cdot wx_\alpha + r_\alpha \cdot hx_\alpha \le x_\beta + (1 - a_{\alpha\beta}) \cdot M \quad \forall \alpha, \beta,\ \alpha < \beta \quad (17)$$

$$x_\beta + p_\beta \cdot lx_\beta + q_\beta \cdot wx_\beta + r_\beta \cdot hx_\alpha \le x_\alpha + (1 - b_{\alpha\beta}) \cdot M \quad \forall \alpha, \beta,\ \alpha < \beta \quad (18)$$

$$y_\alpha + q_\alpha \cdot wy_\alpha + p_\alpha \cdot ly_\alpha + r_\alpha \cdot hy_\alpha \le y_\beta + (1 - c_{\alpha\beta}) \cdot M \quad \forall \alpha, \beta,\ \alpha < \beta \quad (19)$$

$$y_\beta + q_\beta \cdot wy_\beta + p_\beta \cdot ly_\beta + r_\beta \cdot hy_\beta \le y_\alpha + (1 - d_{\alpha\beta}) \cdot M \quad \forall\, \alpha, \beta,\ \alpha < \beta \tag{20}$$

$$z_\alpha + r_\alpha \cdot hz_\alpha + q_\alpha \cdot wz_\alpha + p_\alpha \cdot lz_\alpha \le z_\beta + (1 - e_{\alpha\beta}) \cdot M \quad \forall\, \alpha, \beta,\ \alpha < \beta \tag{21}$$

$$z_\beta + r_\beta \cdot hz_\beta + q_\beta \cdot wz_\beta + p_\beta \cdot lz_\beta \le z_\alpha + (1 - f_{\alpha\beta}) \cdot M \quad \forall\, \alpha, \beta,\ \alpha < \beta \tag{22}$$

$$a_{\alpha\beta} + b_{\alpha\beta} + c_{\alpha\beta} + d_{\alpha\beta} + e_{\alpha\beta} + f_{\alpha\beta} \ge s_{\alpha k} + s_{\beta k} - 1 \quad \forall\, \alpha, \beta, k,\ \alpha < \beta \tag{23}$$

$$x_\alpha + p_\alpha \cdot lx_\alpha + q_\alpha \cdot wx_\alpha + r_\alpha \cdot hx_\alpha \le L_k + (1 - s_{\alpha k}) \cdot M \quad \forall\, \alpha, k \tag{24}$$

$$y_\alpha + q_\alpha \cdot wy_\alpha + p_\alpha \cdot ly_\alpha + r_\alpha \cdot hy_\alpha \le W_k + (1 - s_{\alpha k}) \cdot M \quad \forall\, \alpha, k \tag{25}$$

$$z_\alpha + r_\alpha \cdot hz_\alpha + q_\alpha \cdot wz_\alpha + p_\alpha \cdot lz_\alpha \le H_k + (1 - s_{\alpha k}) \cdot M \quad \forall\, \alpha, k \tag{26}$$

$$\sum_{\alpha \in K_i} s_{\alpha k} - n_i \cdot \sum_j \psi_{ijk} = 0 \quad \forall\, i, k \tag{27}$$

$$lx_\alpha + ly_\alpha + lz_\alpha = 1 \tag{28}$$

$$wx_\alpha + wy_\alpha + wz_\alpha = 1 \tag{29}$$

$$hx_\alpha + hy_\alpha + hz_\alpha = 1 \tag{30}$$

$$lx_\alpha + wx_\alpha + hx_\alpha = 1 \tag{31}$$

$$ly_\alpha + wy_\alpha + hy_\alpha = 1 \tag{32}$$

$$lz_\alpha + wz_\alpha + hz_\alpha = 1 \tag{33}$$

$$\psi_{ijk} \in \{0,1\} \quad \forall\, i, j, k \tag{34}$$

$$u_i \ge 0 \quad \text{and integer} \quad \forall\, i \in N \tag{35}$$

$$lx_\alpha, ly_\alpha, lz_\alpha, wx_\alpha, wy_\alpha, wz_\alpha, hx_\alpha, hy_\alpha, hz_\alpha \in \{0,1\} \quad \text{and} \quad x_\alpha, y_\alpha, z_\alpha \ge 0 \quad \forall\, \alpha \tag{36}$$

$$a_{\alpha\beta}, b_{\alpha\beta}, c_{\alpha\beta}, d_{\alpha\beta}, e_{\alpha\beta}, f_{\alpha\beta} \in \{0,1\} \quad \forall\, \alpha, \beta \quad \text{and} \quad s_{\alpha k} \in \{0,1\} \quad \forall\, \alpha, k \tag{37}$$

The goal function, which involves both route costs and volume utilization, is identified in (10). Total distance is minimized for routing, while unused volume is minimized for packing. Goals are merged with two parameters μ and γ. Values of these parameters may be varied due to application priorities, satisfying the equation $\mu + \gamma = 1$. γ may have higher values in situations that volume utilization is important and μ may have higher values in situations that route costs are important. It is reasonable to take parameters as $\mu > \gamma$, since the route cost savings are generally more important than that obtained from volume utilization.

Constraints between (11)-(16) identify the vehicle routing problem. (15) declares vehicle capacities by weights. Constraints between (17)-(22) determine mutual positionings of the loads in a vehicle. The “mutual positioning” condition of the loads is valid only if the loads are in the same vehicle and this is declared by (23). Constraints between (24) and (26) impose vehicle container dimensions.

The equation in (27) makes sure that the boxes of a city are loaded into the vehicle that visits the city. For example, let $K_1 = \{a, b, c\}$ be the set of the boxes belonging to city "1". Having $|K_1| = n_1 = 3$, (27) can be stated as below for city "1".

$$s_{ak} + s_{bk} + s_{ck} = 3 \cdot \sum_j \psi_{1jk} \quad \forall k$$

In this way, variables $s_{\alpha k}$ related with boxes "a,b,c" are enforced to take value of "1", for the vehicle visiting the city "1".

Constraints between (28)-(33) are logical positioning statements. (34)-(37) identify variable conditions. The model can be simplified by deleting the statements between (28)-(33) and performing the transformations: "$ly_\alpha = 1 - lx_\alpha - lz_\alpha$", "$hx_\alpha = 1 - lx_\alpha - lz_\alpha + wy_\alpha - hz_\alpha$", "$hy_\alpha = lx_\alpha + lz_\alpha - wy_\alpha$", "$wx_\alpha = lz_\alpha - wy_\alpha + hz_\alpha$" and "$wz_\alpha = 1 - lz_\alpha - hz_\alpha$".

Results of the model

Model is applied to a problem where loads four cities will be collected and brought back to the depot. Coordinates of the cities and corresponding load data are presented in Table 3. Vehicle and cost (distance) data can be seen in Table 4 and 5 respectively. Negligible values are used for weights of boxes in statement (15). Distance values are rounded and multiplied by ten to get the real scale. Geographical dispersion of the cities can be seen in Figure 3_a). Though the cities have one load each in the problem, the model is valid in the case that any city has more than one load.

If the problem is handled with classical VRP model, (38) will be valid by the expression in (6). Since $72 + 450 + 210 + 105 \leq 900$, the first vehicle can visit all of the cities. The route obtained from the solution of the model is presented in Figure 3_b). According to the solution, only the first vehicle will be used.

$$72 \cdot \sum_j x_{1j1} + 450 \cdot \sum_j x_{2j1} + 210 \cdot \sum_j x_{3j1} + 105 \cdot \sum_j x_{4j1} \leq 900 \qquad (38)$$

Table 3. City and load data for the VRPPC example.

City	Coordinates		Load Dimensions			Volume
i	**x(*i*)**	**y(*i*)**	**L (dm)**	**W (dm)**	**H (dm)**	**(m^3)**
0	57.47	11.80	-	-	-	-
1	19.68	40.63	60	60	20	72
2	26.13	87.03	100	90	50	450
3	70.37	93.33	70	60	50	210
4	80.97	42.43	70	50	30	105

Table 4. Vehicle data for the VRPPC example.

Vehicle	L (dm)	W (dm)	H (dm)	Capacity (m^3)
1	180	100	50	900
2	70	60	30	126

Table 5. Cost (distance) data for the VRPPC example.

	0	**1**	**2**	**3**	**4**
0	-	480	810	830	390
1	480	-	470	730	610
2	810	470	-	450	710
3	830	730	450	-	520
4	390	610	710	520	-

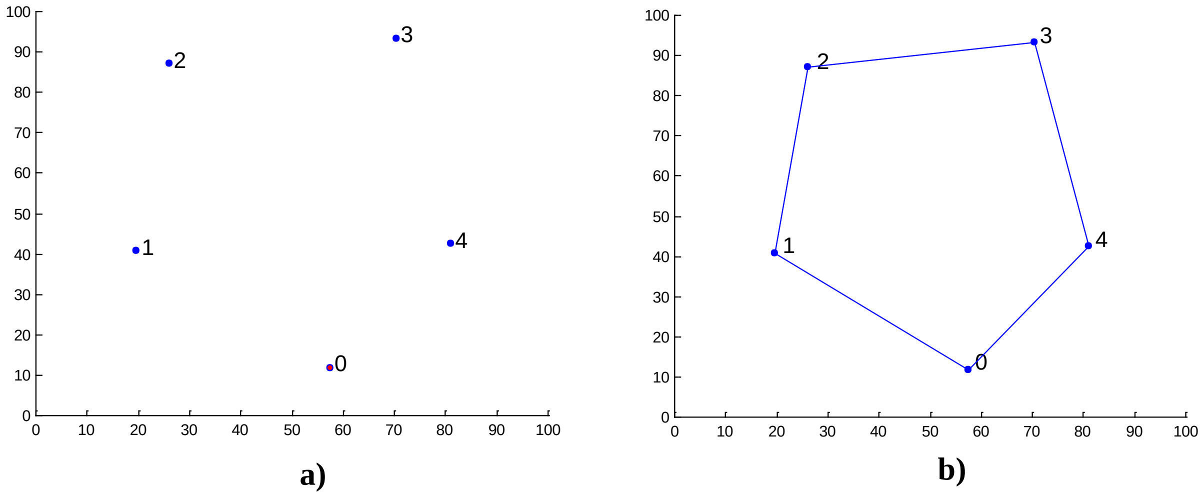

Figure 3. a) Geographical dispersion of the cities. b) VRP solution (0–1–2–3–4–0).

After the vehicle starts to collect the loads, probable loading scheme will be like in Figure 4_a). It can be seen that there is not enough space in the vehicle for "$70 \times 60 \times 50$" dimensional load of the third city. Even if the vehicle collects the load of fourth city (Figure 4_b), the second vehicle to be assigned to third city is small dimensional and so it will not be possible for this vehicle to take the load of the city. Consequently, obtained route from VRP model is void and can not be used for this operation.

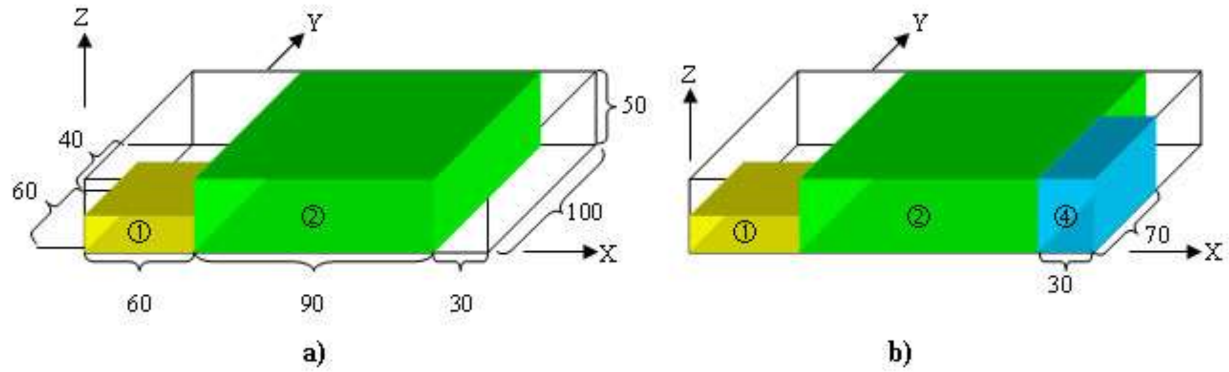

Figure 4. Probable loading scheme for the VRP solution: a) Loads of cities "1" and "2" b) Loads of cities "1", "2" and "4".

As seen in the example, classical VRP approach is incapable in the case of transportation of unit loads. When the problem is solved with the developed VRPPC model, results are obtained as in Table 6. Parameters $\mu = 0.8$ and $\gamma = 0.2$ are used for solution. Routes obtained from the model can be seen in Figure 5, which are: "0 – 4 – 3 – 2 – 0" for the first vehicle and "0 – 1 – 0" for the second vehicle.

Table 6. Results of the VRPPC model for the sample problem.

Box	**Coordinates**			**Positioning**			**Couple**	**Mutual Positionings**					
i	x	y	z	L…	W…	H…	(i,k)	a_{ik}	b_{ik}	c_{ik}	d_{ik}	e_{ik}	f_{ik}
1	0	0	0	LY	WX	HZ	2	1	0	0	0	0	0
							3	0	0	0	0	0	0
							4	1	0	0	0	0	0
2	60	0	0	LY	WX	HZ	3	0	1	0	0	0	0
							4	1	0	0	0	0	0
3	0	0	0	LY	WX	HZ	4	1	0	0	0	0	0
4	150	0	0	LY	WZ	HX	-	-	-	-	-	-	-

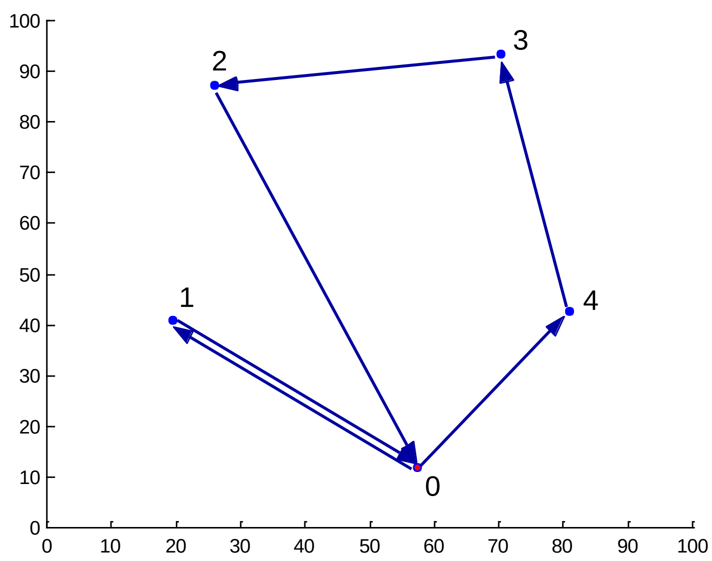

Figure 5. Routes obtained from solution of VRPPC model.

Loading schemes seen in Figure 6 are obtained from the data in Table 6. Having the FLB coordinates of the boxes, placements are performed according to binary decision variables. For example, FLB coordinates of the box "2" are "(60, 0, 0)" and the related variable values are "$ly_2 = wx_2 = hz_2 = 1$". According to these values length, width and height of the box is parallel to "*y*", "*x*" and "*z*" axes respectively.

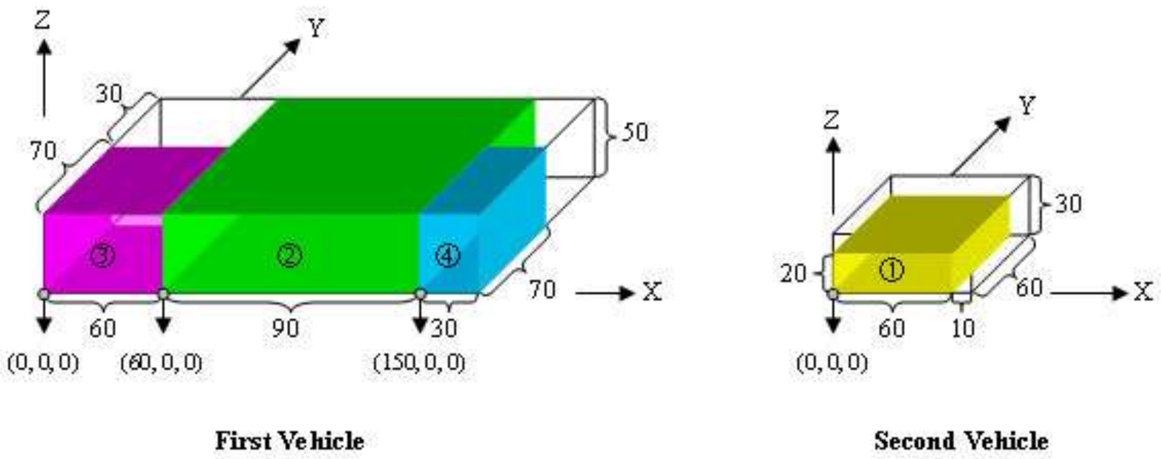

Figure 6. Loading schemes obtained from VRPPC model.

Boxes "2", "3" and "4" are in the *first vehicle* and box "1" is in the *second vehicle*. Mutual positioning variable values of box "1" are not important, since this box is not in the same vehicle with the others. Values: "$b_{23} = a_{24} = a_{34} = 1$" indicate that, box "2" is in the right of box "3" and in the left of box "4". Box "3" is in the left of box "4".

Conclusion

In this paper, a new model for the vehicle routing problem is developed which involves packing conditions for transportation of unit loads. If loads with identified dimensions are in question, packing constraints should be considered in vehicle routing model in order to get valid routes. To the knowledge of the author, no vehicle routing model with packing constraints has been developed up to date.

By integrating packing constraints, vehicle routing problem is getting rid of its overlooked deficiency and is gaining a new insight for future studies. This approach has direct impact on routing applications and it can be thought as *a new opportunity for operations research* as well as being a new opportunity for the routing area.

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